Eur. Phys. J. C 23, 463–469 (2002)
Digital Object Identifier (DOI) 10.1007/s100520200903

THE EUROPEAN
PHYSICAL JOURNAL C

Results for all reactions $e^+e^- \to 4\mathrm{f}, 4\mathrm{f}\gamma$ with nonzero fermion masses*

F. Jegerlehner[1], K. Kołodziej[2]

[1] Deutsches Elektronen-Synchrotron DESY, Platanenallee 6, 15738 Zeuthen, Germany
[2] Institute of Physics, University of Silesia, ul. Uniwersytecka 4, 40007 Katowice, Poland

Received: 2 October 2001 / Revised version: 2 January 2002 /
Published online: 1 March 2002 –

Abstract. We accomplish our efforts to obtain predictions for all four–fermion final states of e^+e^-–annihilation and the corresponding bremsstrahlung reactions which are possible in the framework of the Standard Model. For this purpose we have developed a program `ee4f`γ. Our predictions are valid for fermions of arbitrary masses and we can obtain results for total cross sections without any collinear cut. Keeping exact fermion masses is of course required for top quark production. We give a detailed phenomenological analysis of fermion mass effects and real photon radiation for all channels of four–fermion production at LEP-II and next linear collider energies.

1 Introduction

Among the most attractive options of facilities at the high energy frontier of elementary particle physics are high luminosity e^+e^- linear colliders like TESLA [1,2], the NLC [3] or the JLC [4]. However, going to higher energies and higher luminosity becomes a real challenge for working out Standard Model (SM) predictions of the adequate precision because of the dramatically increasing complexity of perturbative calculations. Here we consider all four fermion production processes in electron positron annihilation together with real photon emission at the tree level. These processes with 6 and 7 external particles at the tree level are described by from 10 up to 1008 Feynman diagrams in a given channel, neglecting the Higgs boson coupling to light fermion flavors, and the physical cross section is the result of a very obscure quantum mechanical interference between all these diagrams. Interesting are of course those cases where the result is dominated by a few diagrams like in W–pair production where we have three relevant "signal diagrams". However, also in these cases which allow a relatively simple physical interpretation, many other diagrams may play a role as a background contribution which affects the precise interpretation of the signal process. The latter give us information about the gauge boson parameters and the triple or quadruple gauge couplings. Similarly, the properties of the Higgs boson will be fixed by its contribution as unstable intermediate state. The most important physics cases have been reviewed in [2], for example. A number of dominant $e^+e^- \to 4\mathrm{f}$ channels have been explored experimentally at LEP-II (1996-2000) [5] and the measurements confirmed SM predictions at the level of the most relevant $O(\alpha)$ corrections.

In the approximation of massless fermions all possible four fermion channels $e^+e^- \to 4\mathrm{f}$ have been investigated in [6] (`EXCALIBUR`) and those for $e^+e^- \to 4\mathrm{f}\gamma$ in [7] (`RacoonWW`). Here we extend these investigations to a calculation with nonzero fermion masses. Keeping nonzero fermion masses will be important in cases where predictions at the 1% accuracy level are required [8,9]. Finite masses also provide a natural regularization of distributions which become singular in the massless limit. Massive calculations thus provide reliable benchmarks for massless calculations with cuts. The latter are much simpler and hence much faster than calculations with massive codes. The hard bremsstrahlung processes are of interest in their own right and may be used to investigate anomalous $WW\gamma$ and $WW\gamma\gamma$ couplings, for example.

Our calculation is considered to be a building block (the soft plus hard bremsstrahlung part) for a complete $O(\alpha)$ calculation of the processes $e^+e^- \to 4\mathrm{f}$. Such calculations have been attempted in [10] (see also [11]). This would also extend existing calculations of W–pair production in the double pole approximation [12] (`RacoonWW`) and [13] (`KORALW/YFSWW`) (see also [14] (`EEWW`)) which incorporate the one-loop corrections for production of on–shell W–pairs [15] and their subsequent decay into fermion–pairs [16].

There already exist a number of codes which allow to calculate exact matrix elements for $e^+e^- \to 4\mathrm{f}$, $4\mathrm{f}\gamma$ for massive fermions. Some of the program packages available

* Work supported in part by the Polish State Committee for Scientific Research (KBN) under contract No. 2 P03B 004 18 and by European Commission's 5-th Framework contract HPRN-CT-2000-00149

are general purpose packages which allow for an automatic calculation of tree–level amplitudes and for their numerical evaluation. Known programs, which may be utilized for tree-level calculations of the kind we are interested in are: `GRACE/BASES` [17], `MADGRAPH/HELAS` [18], `CompHEP` [19] (squared matrix element technique), `WPHACT` [20], `NEXTCALIBUR` [21] (initial state radiation photons generated via the structure function approach), `HELAC/PHEGAS` [22] (recursive Dyson-Schwinger equation approach), `WRAP` [23] (`ALPHA` [24] algorithm) and `O'Mega/WHIZARD` [25]. Most of the codes work on the basis of helicity amplitudes and some use the structure function approach to generate the photons. Except for `NEXTCALIBUR`, which is specialized to $e^+e^- \to 4\mathrm{f}$, all other programs allow to generate and evaluate amplitudes for other type of processes. For more details and comparisons we refer to the *Four Fermion Working Group Report* [26].

Many channels have their own specific problems concerning numerical stability and/or efficiency and need separate consideration and optimization. We therefore present, in this paper, a different approach which is optimized for each individual channel. We present reference tables for cross section calculations at $\sqrt{s} = 200$ GeV and 500 GeV for all leptonic, semi–leptonic and hadronic channels. The present work completes previous investigations of specific channels presented in [8,9].

Our calculation has to be extended to include the $O(\alpha)$ virtual corrections in future. Precise knowledge of the various channels is crucial for the precise determination of properties of the unstable gauge and Higgs bosons as well as to reveal possible anomalous coupling [2,27,28] which might exist beyond the SM.

In the following we outline our calculation, present the numerical results and end with the conclusions.

2 Calculation

The matrix elements of the reactions

$$e^+e^- \to 4f \tag{1}$$

and

$$e^+e^- \to 4f\gamma \tag{2}$$

are calculated by utilizing the helicity amplitude method described in [8]. As in [8], the photon propagator is taken in the Feynman gauge while for the propagators of the massive gauge bosons we use the unitary gauge. Constant widths of the electroweak gauge bosons, Γ_W, Γ_Z, Higgs boson, Γ_H, and the top quark, Γ_t are introduced through the complex mass parameters

$$\begin{aligned} &M_V^2 = m_V^2 - i m_V \Gamma_V, \quad V = W, Z, \\ &M_H^2 = m_H^2 - i m_H \Gamma_H, \qquad M_t = m_t - i\Gamma_t/2, \end{aligned} \tag{3}$$

in the corresponding propagators

$$\begin{aligned} &\Delta_F^{\mu\nu}(q) = \frac{-g^{\mu\nu} + q^\mu q^\nu / M_V^2}{q^2 - M_V^2}, \qquad \Delta_F(q) = \frac{1}{q^2 - M_H^2}, \\ &S_F(q) = \frac{\not{q} + M_t}{q^2 - M_t^2}, \end{aligned} \tag{4}$$

both in the s- and t-channel Feynman diagrams. The electroweak mixing parameter is kept real

$$\sin^2\theta_W = 1 - m_W^2/m_Z^2. \tag{5}$$

This kind of parametrization is usually referred to as the fixed-width scheme (FWS). Our results presented in the next section have been obtained in the FWS. In our program `ee4f`γ, it is also possible to define $\sin^2\theta_W$ in terms of the complex masses of (3) as

$$\sin^2\theta_W = 1 - M_W^2/M_Z^2, \tag{6}$$

which is usually called the complex-mass scheme (CMS). The CMS has the advantage that it satisfies the $SU(2) \times U(1)$ Ward identities at tree level [7]. However, the fact that (6) makes some of the SM couplings complex quantities may become a source of discomfort. In the FWS on the other hand, all the couplings remain real. Only the electromagnetic gauge invariance is satisfied exactly, however, and this only provided that Γ_t and the other fermion widths are vanishing. It should be stressed at this point, that for vanishing fermion widths, electromagnetic gauge invariance is preserved with non-zero fermion masses and with the gauge boson widths Γ_W and Γ_Z treated as independent parameters. If a non-vanishing top quark width is introduced through substitution (3), which is done in order to regularize the on-mass-shell pole of a top quark propagator at tree level in reactions (2) containing a single top quark in the final state, the external electromagnetic gauge invariance gets violated. It can be restored by redefining the Dirac bi-spinor representing the external top quark in such a way that it satisfies the Dirac equation with the complex top quark mass of (3), which would obviously require a complex top quark four momentum in the phase space generation. As the production of a on-mass-shell top quark is a rather unphysical process, one should not be to much concerned about the problem. Fortunately, as we shall see in the next section, the violation of the gauge symmetry does not lead to dramatic effects for the total cross sections of reactions (2) containing a top quark in the final state. A more realistic treatment of the top quark has to include its decay and thus requires the consideration of $e^+e^- \to 6f, 6f\gamma$ channels, which is beyond the task of the present investigation.

The hadronic channels are discussed only at the level of quark parton production. Quark-mass effects will be estimated by adopting the so–called current-quark masses in the $\overline{\mathrm{MS}}$ scheme at a scale $\mu \sim 2$ GeV. This should allow us to get an idea about the size of mass effects and eventually allow us to establish suitable cuts which eliminate the mass sensitivity of quark production cross sections. In any case, taking into account mass effects, provides an improvement over calculations in the approximation of massless quarks. For observables which exhibit a substantial mass dependence of course one would have to discuss more carefully the precise physical meaning of quark masses in the given process.

3 Results

In this section, we will present numerical results for all the four–fermion channels of reactions (1) and (2) which are possible in the SM.

We define the SM physical parameters in terms of the gauge boson masses and widths, the top mass and width, and the Fermi coupling constant. We take the actual values of the parameters from [29]:

$$m_W = 80.419 \text{ GeV}, \quad \Gamma_W = 2.12 \text{ GeV},$$
$$m_Z = 91.1882 \text{ GeV}, \quad \Gamma_Z = 2.4952 \text{ GeV},$$
$$m_t = 174.3 \text{ GeV}, \quad G_\mu = 1.16639 \times 10^{-5} \text{ GeV}^{-2}. \tag{7}$$

We assume a Higgs boson mass of $m_H = 115$ GeV and calculate the Higgs boson width with the lowest order formula in the SM. The top quark width is assumed to be $\Gamma_t = 1.5$ GeV.

For the sake of definiteness we also list the other fermion masses that we use in the calculation [29]:

$$m_e = 0.510998902 \text{ MeV}, \quad m_\mu = 105.658357 \text{ MeV},$$
$$m_\tau = 1777.03 \text{ MeV}, \quad m_u = 5 \text{ MeV}, \quad m_d = 9 \text{ MeV},$$
$$m_s = 150 \text{ MeV}, \quad m_c = 1.3 \text{ GeV}, \quad m_b = 4.4 \text{ GeV}. \tag{8}$$

We neglect the Cabibbo–Kobayashi–Maskawa (CKM) mixing, i.e., we assume the CKM matrix to be the unit matrix. However, it is possible to run the program with nontrivial CKM mixing as well.

The effective fine structure constant (at scale $\sim M_W$) is calculated via

$$\alpha_W = \sqrt{2} G_\mu m_W^2 \sin^2\theta_W/\pi \tag{9}$$

utilizing the real electroweak mixing parameter $\sin^2\theta_W$ defined by (5). In `ee4f`γ, it is also possible to perform computations with the complex $\sin^2\theta_W$ of (6) and the complex m_W^2 of (3), i.e. with the complex α_W. The photon coupling to fermions and gauge bosons is given by the fine structure constant in the Thomson limit $\alpha = 1/137.0359895$ and the quark–gluon strong interaction "constant" by $\alpha_s(M_Z) = 0.1185$.

We will apply the following set of "standard cuts" which have been proposed in [26]:

$$\cos\theta(l, \text{beam}) \le 0.985, \ \theta(\gamma, l) > 5^\circ, \ E_\gamma > 1 \text{ GeV},$$
$$m(q, q') > 10 \text{ GeV},$$
$$\cos\theta(\gamma, \text{beam}) \le 0.985, \ \theta(\gamma, q) > 5^\circ, \ E_l > 5 \text{ GeV}, \tag{10}$$

where l, q, γ, and "beam" denote charged leptons, quarks, photons, and the beam (electrons or positrons), respectively, and $\theta(i,j)$ the angles between the particles i and j in the center of mass system. Furthermore, $m(q,q')$ denotes the invariant mass of a quark pair qq'. Note that we are not applying a corresponding cut to the invariant mass of the charged lepton pairs.

The errors we will quote in the Tables below have been evaluated as follows: For each separate channel of the multichannel Monte Carlo (MC) integration the error is calculated by `VEGAS` [30]. This is a purely statistical error equivalent to one standard deviation. We added linearly standard deviations for all the channels used in an integration and this is what is our error. This provides a more conservative estimate for the error than for example adding up partial errors in quadrature.

Except for the check of electromagnetic gauge invariance, discussed in the previous section, we perform a few other checks. Whenever the fermion masses play no role we have reproduced the results of [7]. The matrix elements of almost all channels of the processes (1) and (2) under consideration have been checked against `MADGRAPH` [18]. The comparison was not simple for the channels involving a gluon exchange, since the version of `MADGRAPH` which we are using, generates either the electroweak or the QCD part, but not both simultaneously. In addition, for the reaction $e^+e^- \to e^+e^-e^+e^-\gamma$ `MADGRAPH` generates only 999 instead of all 1008 Feynman graphs. The phase space generation routines have been thoroughly checked against each other before they have been combined into a multichannel phase space generation routine. The total cross sections of the reactions (1) containing a single top quark in the final state

$$e^+e^- \to t\bar{b}f\bar{f}', \tag{11}$$

where $f = e^-, \mu^-, \tau^-, d, s$ and $f' = \nu_e, \nu_\mu, \nu_\tau, u, c$, respectively, have been calculated in an arbitrary linear gauge [31]. This does not allow to estimate the absolute size of gauge violation effects caused by the nonzero widths of the unstable fermions. However, as the transition between two linear gauges, the 't Hooft–Feynman and unitary gauge, which has been numerically performed by changing the gauge parameter from 1 to 10^{16} has caused practically negligible change in the cross sections at center of mass energies up to 2 TeV, typical for a linear collider, one may expect that the gauge violation effects are not very dramatic for total cross sections.

Moreover, another test of reliability of our results has been performed for several final states. We have split the cross section of the bremsstrahlung process (2) into a soft photon part σ_s, which includes the photons with energies $E_\gamma \le \omega$, and a hard photon part σ_h, including the contributions from photon with energies $E_\gamma > \omega$, and checked whether the combined bremsstrahlung cross section $\sigma_\gamma = \sigma_s + \sigma_h$ is independent of the photon energy cut ω [8]. In Table 1, we illustrate this independence, which holds within one standard deviation of the MC integration, for $e^+e^- \to c\bar{s}\tau^-\bar{\nu}_\tau\gamma$ in the CMS. At the same time, in Table 1, we see a small dependence on the cut–off parameter ω, which is at the level of about two standard deviations, for the bremsstrahlung reaction $e^+e^- \to t\bar{b}\tau^-\bar{\nu}_\tau\gamma$. The cut dependence is most probably induced by the violation of external electromagnetic gauge invariance caused by the nonzero top quark width, which has been introduced to $e^+e^- \to t\bar{b}\tau^-\bar{\nu}_\tau\gamma$ in a somewhat asymmetric way, related to the fact that the top quark is regarded as on-mass-shell particle at the same time when the anti-top quark decays. It should be stressed, that the soft bremsstrahlung cross sections σ_s presented in Table 1 are unphysical, as they contain the unphysical photon mass m_γ. They are only given in order to show that

Table 1. Cross sections in femto-barns (1 fb=10^{-15} barns) of $e^+e^- \to c\bar{s}\tau^-\bar{\nu}_\tau\gamma$ and $e^+e^- \to t\bar{b}\tau^-\bar{\nu}_\tau\gamma$ in CMS at $\sqrt{s} = 500$ GeV for different photon energy cuts ω. σ_s, corresponding to $E_\gamma \leq \omega$, is the cross section in the soft photon limit and σ_h, corresponding to $E_\gamma > \omega$, is the hard bremsstrahlung cross section. A fictitious photon mass $m_\gamma = 10^{-6}$ GeV has been introduced in order to regularize the infrared divergence. No other cuts except for ω and m_γ are present

ω	$e^+e^- \to c\bar{s}\tau^-\bar{\nu}_\tau\gamma$			$e^+e^- \to t\bar{b}\tau^-\bar{\nu}_\tau\gamma$		
(GeV)	σ_s (fb)	σ_h (fb)	$\sigma_s + \sigma_h$ (fb)	σ_s (fb)	σ_h (fb)	$\sigma_s + \sigma_h$ (fb)
0.1	186.37(9)	250.7(3)	437.1	51.85(3)	56.88(9)	108.7
0.01	108.89(6)	328.1(3)	437.0	32.29(2)	76.1(1)	108.4
0.001	31.39(2)	405.5(4)	436.9	12.729(8)	95.3(1)	108.0

Table 2. Cross sections in fb of $e^+e^- \to c\bar{s}\mu^-\bar{\nu}_\mu$ at $\sqrt{s} = 200$ GeV for different cuts on the photon angle with respect to the quarks $\theta(\gamma, q)$ or muon $\theta(\gamma, \mu)$ and the remaining cuts as in (10). Here we use physical parameters of [23] and parametrize the photon couplings by $\alpha = 1/137.0359895$

$\theta(\gamma, q)$	$\theta(\gamma, \mu)$	[23]	Present work
$5°$	$5°$	74.294(29)	74.267(60)
$1°$	$1°$	93.764(37)	93.70(7)
$5°$	$1°$	90.157(36)	90.13(7)
$5°$	$0.1°$	104.777(46)	104.78(7)
$5°$	$0°$	105.438(45)	105.48(7)

the leading logarithmic contributions are treated properly within the CMS for reactions which do not contain nonzero fermion width and to illustrate the size of cut off dependence caused by the nonzero top quark width. As the cut dependence in $e^+e^- \to t\bar{b}\tau^-\bar{\nu}_\tau\gamma$ is of the order of 1% of the corresponding four fermion Born cross section, one should certainly elaborate more on this issue in future, in the context of a more realistic six fermion reactions, which would treat the decay of the top quark on the same footing as that of the anti-top quark.

We compare our results for the total cross sections of $e^+e^- \to c\bar{s}\mu^-\bar{\nu}_\mu$ at $\sqrt{s} = 200$ GeV with those of [23] in Table 2. As in [23], different cuts on the photon angle with respect to the quarks $\theta(\gamma, q)$ or muon $\theta(\gamma, \mu)$ are imposed while the remaining cuts are those given in (10). For the comparison we use the physical parameters of [23], i.e. $G_\mu = 1.16637 \times 10^{-5}$ GeV^{-2}, $m_Z = 91.1867$ GeV, $m_W = 80.35$ GeV, $\sin^2\theta_W = 1 - m_W^2/m_Z^2$, $\Gamma_Z = 2.49471$ GeV, $\Gamma_W = 2.04277$ GeV, $m_\mu = 0.10565839$ GeV, $m_s = 0.15$ GeV, $m_c = 1.55$ GeV. Although it has not been explicitly specified, we assume that [23] is using $\alpha = 1/137.0359895$ for the photon coupling strength. We find that the results agree perfectly within one standard deviation of the MC integration.

Our results for all channels of reactions $e^+e^- \to 4f$, $4f\gamma$ possible in the SM are collected in Tables 3–7. We present total cross sections at two center of mass energies, $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV, with cuts defined by (10), except for $e^+e^- \to e^+e^-e^+e^-$, where we have imposed another cut on the angle between the final state electrons and/or positrons, $\theta(e^\pm, e^\pm) > 5°$.

Table 3. $e^+e^- \to 4f$ cross sections σ and $e^+e^- \to 4f\gamma$ cross sections σ_γ in fb at $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV for different four–fermion final states corresponding to the W^+W^-–pair signal. The cuts are those of (10)

Final	$\sqrt{s} = 200$ GeV		$\sqrt{s} = 500$ GeV	
state	σ	σ_γ	σ	σ_γ
$u\bar{d}\mu^-\bar{\nu}_\mu$	630.65(31)	70.547(83)	211.11(13)	23.601(46)
$u\bar{d}\tau^-\bar{\nu}_\tau$	630.18(31)	68.321(74)	210.95(13)	23.386(44)
$c\bar{s}\mu^-\bar{\nu}_\mu$	630.40(31)	69.501(80)	211.03(13)	23.285(47)
$c\bar{s}\tau^-\bar{\nu}_\tau$	629.93(31)	67.279(72)	210.87(13)	23.077(43)
$t\bar{b}\mu^-\bar{\nu}_\mu$	–	–	58.88(30)	9.467(60)
$t\bar{b}\tau^-\bar{\nu}_\tau$	–	–	58.80(29)	9.295(56)
$c\bar{s}d\bar{u}$	1838.6(1.4)	172.74(28)	749.07(50)	68.34(23)
$t\bar{b}d\bar{u}$	–	–	177.8(1.9)	25.55(42)
$t\bar{b}s\bar{c}$	–	–	177.4(1.9)	25.37(37)
$\nu_\tau\tau^+\mu^-\bar{\nu}_\mu$	205.88(15)	25.784(44)	60.762(62)	7.842(23)
$u\bar{d}d\bar{u}$	1921.4(7)	188.19(46)	780.66(25)	74.99(28)
$c\bar{s}s\bar{c}$	1925.7(8)	184.07(46)	782.62(28)	73.46(25)
$t\bar{b}b\bar{t}$	–	–	0.85519(56)	0.073748(78)
$\nu_\mu\mu^+\mu^-\bar{\nu}_\mu$	218.91(19)	28.232(55)	63.933(70)	8.475(26)
$\nu_\tau\tau^+\tau^-\bar{\nu}_\tau$	214.94(20)	26.280(50)	63.468(74)	8.299(20)
$\nu_e e^+e^-\bar{\nu}_e$	259.55(31)	32.012(93)	195.22(42)	24.85(14)

In Tables 3 and 4, we show the results for the channels corresponding to the W^+W^-–pair and single–W signal. These channels are usually classified as charged current reactions. The relative magnitude of the cross sections in both tables reflects the naive counting of the color degrees of freedom, e.g., the cross sections of purely hadronic channels are about a factor 3 bigger than the cross sections of semi-leptonic channels and the latter are a factor 3 bigger than the cross sections of purely leptonic reactions. This somewhat general rule is obviously violated in reactions which receive contributions from the gluon or t-channel photon and Z exchange. Except for the channels containing heavy quarks, t and b, for lighter flavors, the fermion mass effects are not big. However, for individual channels they are of the order of a few per cent, as it has been al-

Table 4. $e^+e^- \to 4\mathrm{f}$, $4\mathrm{f}\gamma$ cross sections in fb at $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV for different four–fermion final states corresponding to the single–W signal. The cuts are those of (10)

Final	$\sqrt{s} = 200$ GeV		$\sqrt{s} = 500$ GeV	
state	σ	σ_γ	σ	σ_γ
$u\bar{d}e^-\bar{\nu}_e$	661.68(40)	72.95(10)	354.06(27)	38.876(88)
$c\bar{s}e^-\bar{\nu}_e$	661.42(40)	71.84(10)	353.91(27)	38.371(88)
$t\bar{b}e^-\bar{\nu}_e$	–	–	58.54(65)	9.43(16)
$\nu_\mu\mu^+e^-\bar{\nu}_e$	216.29(21)	27.473(57)	107.34(14)	13.677(43)
$\nu_\tau\tau^+e^-\bar{\nu}_e$	216.13(21)	26.709(55)	107.25(13)	13.471(42)

Table 5. Cross sections in fb of the purely leptonic neutral-current channels of (1) and (2) at $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV. The cuts are given by (10) with the exception of $e^+e^- \to e^+e^-e^+e^-$, where we have imposed another cut on the angle between the final state electrons and/or positrons, $\theta(e^\pm, e^\pm) > 5°$

Final	$\sqrt{s} = 200$ GeV		$\sqrt{s} = 500$ GeV	
state	σ	σ_γ	σ	σ_γ
$\mu^+\mu^-\tau^+\tau^-$	10.267(14)	2.1787(91)	2.5117(44)	0.6495(40)
$\mu^+\mu^-\bar{\nu}_\tau\nu_\tau$	12.729(10)	1.6998(44)	3.0336(31)	0.5695(20)
$\tau^+\tau^-\bar{\nu}_\mu\nu_\mu$	9.1659(59)	1.2307(27)	2.7174(27)	0.5221(15)
$\bar{\nu}_\tau\nu_\tau\bar{\nu}_\mu\nu_\mu$	10.608(6)	0.57713(82)	3.9179(43)	0.4645(11)
$\mu^+\mu^-e^+e^-$	137.18(90)	12.93(31)	43.80(38)	4.58(12)
$\tau^+\tau^-e^+e^-$	54.49(18)	7.115(55)	16.860(61)	2.685(23)
$\mu^+\mu^-\bar{\nu}_e\nu_e$	17.780(18)	2.1467(43)	24.031(92)	3.479(20)
$\tau^+\tau^-\bar{\nu}_e\nu_e$	11.721(10)	1.4797(24)	21.389(90)	3.432(21)
$\bar{\nu}_\mu\nu_\mu\bar{\nu}_e\nu_e$	11.448(8)	0.6033(7)	24.728(47)	2.349(7)
$\bar{\nu}_\mu\nu_\mu e^+e^-$	23.503(22)	2.7789(68)	9.453(14)	1.3076(57)
$\mu^+\mu^-\mu^+\mu^-$	6.747(13)	1.4307(84)	1.5106(36)	0.3860(30)
$\tau^+\tau^-\tau^+\tau^-$	3.7283(30)	0.7943(21)	1.0341(11)	0.2727(8)
$\bar{\nu}_\mu\nu_\mu\bar{\nu}_\mu\nu_\mu$	5.2660(23)	0.28562(25)	1.9577(15)	0.23149(34)
$e^+e^-e^+e^-$	50.53(10)	5.770(58)	13.927(32)	2.163(21)
$\bar{\nu}_e\nu_e\bar{\nu}_e\nu_e$	5.9815(27)	0.30563(24)	22.482(61)	2.0856(92)

Table 6. Cross sections in fb of the semi-leptonic neutral-current channels of (1) and (2) at $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV. The cuts are those specified by (10)

Final	$\sqrt{s} = 200$ GeV		$\sqrt{s} = 500$ GeV	
state	σ	σ_γ	σ	σ_γ
$\bar{u}u\mu^+\mu^-$	27.341(30)	4.874(13)	6.855(10)	1.5630(62)
$\bar{u}u\tau^+\tau^-$	19.543(17)	3.4694(98)	5.9096(69)	1.3614(49)
$\bar{u}u\bar{\nu}_\mu\nu_\mu$	22.614(14)	2.2049(34)	8.3860(99)	1.2897(30)
$\bar{c}c\mu^+\mu^-$	27.287(27)	4.826(19)	6.9946(88)	1.5924(74)
$\bar{c}c\tau^+\tau^-$	19.560(16)	3.4407(92)	6.0566(68)	1.3922(48)
$\bar{c}c\bar{\nu}_\mu\nu_\mu$	22.655(13)	2.1695(44)	8.6652(83)	1.3434(39)
$\bar{t}t\mu^+\mu^-$	–	–	0.1832(2)	0.02165(5)
$\bar{t}t\tau^+\tau^-$	–	–	0.11991(10)	0.01554(2)
$\bar{t}t\bar{\nu}_\mu\nu_\mu$	–	–	0.07010(2)	0.004577(4)
$\bar{d}d\mu^+\mu^-$	29.541(24)	4.229(13)	7.0683(80)	1.3715(60)
$\bar{d}d\tau^+\tau^-$	21.256(16)	3.0660(51)	6.2854(61)	1.2494(26)
$\bar{d}d\bar{\nu}_\mu\nu_\mu$	24.634(15)	1.5960(28)	9.0094(91)	1.1230(33)
$\bar{s}s\mu^+\mu^-$	29.542(25)	4.240(14)	7.0702(73)	1.3737(55)
$\bar{s}s\tau^+\tau^-$	21.257(16)	3.0666(76)	6.2907(62)	1.2489(38)
$\bar{s}s\bar{\nu}_\mu\nu_\mu$	24.637(15)	1.5975(29)	9.0125(89)	1.1301(29)
$\bar{b}b\mu^+\mu^-$	29.536(23)	4.150(13)	8.6899(74)	1.7290(56)
$\bar{b}b\tau^+\tau^-$	21.349(16)	3.0179(76)	7.9184(66)	1.5972(43)
$\bar{b}b\bar{\nu}_\mu\nu_\mu$	25.019(15)	1.5457(28)	12.322(9)	1.5911(34)
$\bar{u}ue^+e^-$	77.31(26)	10.438(74)	28.37(11)	4.432(33)
$\bar{c}ce^+e^-$	77.02(19)	10.321(61)	29.030(91)	4.532(32)
$\bar{t}te^+e^-$	–	–	172.63(30)	15.657(45)
$\bar{d}de^+e^-$	58.759(82)	7.308(27)	23.668(46)	3.363(21)
$\bar{s}se^+e^-$	58.734(80)	7.337(26)	23.702(46)	3.377(18)
$\bar{b}be^+e^-$	58.082(66)	7.125(21)	29.38(11)	4.152(27)
$\bar{u}u\bar{\nu}_e\nu_e$	25.502(19)	2.4289(41)	49.18(22)	6.300(38)
$\bar{c}c\bar{\nu}_e\nu_e$	25.907(21)	2.4210(38)	55.13(26)	7.180(47)
$\bar{t}t\bar{\nu}_e\nu_e$	–	–	0.07400(4)	0.004810(4)
$\bar{d}d\bar{\nu}_e\nu_e$	26.820(20)	1.6916(23)	57.03(27)	5.731(36)
$\bar{s}s\bar{\nu}_e\nu_e$	26.824(22)	1.6955(24)	57.320(28)	5.777(41)
$\bar{b}b\bar{\nu}_e\nu_e$	31.346(28)	1.8819(33)	128.31(65)	12.59(12)

ready pointed out in [9]. It is amazing that the mass effect is inverse for $e^+e^- \to u\bar{d}d\bar{u}$ and $e^+e^- \to c\bar{s}s\bar{c}$. The inversion is not due to the Higgs boson exchange, but in fact is a consequence of the cuts (10) which we imposed. The latter reduce the contribution of the s-channel Feynman diagrams to $e^+e^- \to u\bar{d}d\bar{u}$ to much larger extent than to $e^+e^- \to c\bar{s}s\bar{c}$, because the cuts on the invariant mass of the quark pairs restrict the phase space much more severely for lighter quarks than for heavier ones. Without the cuts, the cross section of $e^+e^- \to u\bar{d}d\bar{u}$ becomes bigger than that of $e^+e^- \to c\bar{s}s\bar{c}$, as expected. We do not show cross sections at $\sqrt{s} = 200$ GeV for reactions containing a t-quark in the final states, as they are negligibly small [31].

The results for the neutral current channels of reactions (1) and (2) are shown in Tables 5, 6 and 7. We list purely leptonic channels in Table 5, semi-leptonic channels in Table 6 and purely hadronic channels in Table 7. The cross sections in Tables 5, 6 and 7 are typically much smaller then those of Tables 3 and 4. Mass effects on the other hand are bigger. The stronger dependence on fermion masses can be explained as follows. The neutral-current reactions are dominated by s-channel Feynman diagrams which contain the propagator of a photon decaying into a fermion pair. This causes a $\sim 1/s_{ff'}$ behavior of the matrix element squared and results in a relatively high sensitivity to the fermion pair threshold $s_{ff'} = (m_f + m_{f'})^2$. There is a relatively big effect for charged lepton pairs $\mu^+\mu^-$, $\tau^+\tau^-$ and much smaller effect for quark pairs, except for $\bar{t}t$ of course. This is due to the fact that there is no cut on the invariant mass of a charged lepton pair in (10). Again we observe an inverse mass effect in some channels, especially those containing

Table 7. Cross sections in fb of the purely hadronic neutral-current channels of (1) and (2) at $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV. The cuts are those specified by (10)

Final	$\sqrt{s} = 200$ GeV		$\sqrt{s} = 500$ GeV	
state	σ	σ_γ	σ	σ_γ
$\bar{u}u\bar{s}s$	83.63(12)	13.45(13)	32.561(51)	5.891(41)
$\bar{u}u\bar{b}b$	86.565(95)	14.39(11)	38.475(45)	7.004(36)
$\bar{c}c\bar{d}d$	84.41(12)	13.742(76)	33.150(92)	5.969(41)
$\bar{c}c\bar{b}b$	87.209(83)	14.445(99)	39.237(42)	7.149(35)
$\bar{t}t\bar{d}d$	–	–	0.903(1)	0.0801(2)
$\bar{t}t\bar{s}s$	–	–	0.903(1)	0.0803(2)
$\bar{d}d\bar{s}s$	79.658(95)	9.77(11)	29.812(39)	4.159(34)
$\bar{d}d\bar{b}b$	82.486(87)	10.436(86)	37.435(39)	5.363(26)
$\bar{u}u\bar{c}c$	87.06(13)	15.98(12)	35.826(61)	7.581(52)
$\bar{u}u\bar{t}t$	–	–	0.9071(11)	0.09440(21)
$\bar{u}u\bar{u}u$	42.465(45)	7.794(35)	17.410(27)	3.684(19)
$\bar{c}c\bar{c}c$	43.323(44)	7.935(27)	18.076(23)	3.867(14)
$\bar{d}d\bar{d}d$	39.173(32)	4.791(20)	14.758(15)	2.0706(76)
$\bar{s}s\bar{s}s$	39.167(32)	4.810(20)	14.767(15)	2.0769(76)
$\bar{b}b\bar{b}b$	41.667(24)	5.217(14)	22.242(14)	3.2142(67)

a neutrino pair. The cross section of $e^+e^- \to \bar{d}d\bar{\nu}_e\nu_e$ is bigger than that of $e^+e^- \to \bar{u}u\bar{\nu}_e\nu_e$ although the mass of the d-quark is almost twice as big as that of the u-quark. The inverse mass effect is caused by the invariant mass cut $m(q, q') > 10$ GeV of (10), which is more restrictive for lighter fermion pairs than for heavier ones and by the fact that there is neither an invariant mass cut nor an angular cut on a neutrino-pair. We observe that the mass effects depend on cuts. They may be quite different for different choices of cuts.

Whether or not the mass effects will play a role in the analysis of the future data depends mostly on the luminosity of a future linear collider. If we assume a total integrated luminosity of 500 fb^{-1} they will certainly become relevant. Therefore it is better to keep nonzero fermion masses in the calculation, or test the massless fermion generators against a massive one for each given set of kinematical cuts.

If we compare the channels containing a $\bar{b}b$– or $\bar{c}c$–pair with the channels containing lighter quark flavors, we see a clear signal of the Higgs–strahlung reaction $e^+e^- \to ZH$, especially at $\sqrt{s} = 500$ GeV, which exceeds the ZH production threshold for a Higgs boson mass of 115 GeV. In case of $e^+e^- \to \bar{b}b\bar{\nu}_e\nu_e, \bar{c}c\bar{\nu}_e\nu_e$, we see also a signal of a W^+W^- fusion mechanism of the Higgs boson production. For the final state containing a $\bar{b}b$–pair, the signal is visible already at $\sqrt{s} = 200$ GeV and it becomes much more pronounced at $\sqrt{s} = 500$ GeV.

As the threshold energy for $e^+e^- \to \bar{t}t\bar{t}t$ is bigger than 500 GeV, we do not show its cross section in Table 7. However, even at $\sqrt{s} = 800$ GeV the cross section of $e^+e^- \to \bar{t}t\bar{t}t$ without cuts is of the order of 10^{-3} fb.

4 Conclusions

We have developed a program package `ee4f`γ which allows us to calculate in an efficient manner any process $e^+e^- \to 4\mathrm{f},\ 4\mathrm{f}\gamma$ keeping nonzero fermion masses. Corresponding results have been investigated and discussed for all channels at reference energies $\sqrt{s} = 200$ GeV and 500 GeV. For detailed investigations of physics at a high luminosity linear collider like TESLA, these mass effects should be taken into account. Nonzero fermion masses also provide a physical regularization of matrix elements which exhibit collinear singularities in the zero mass limit. The results thus may be used as benchmarks for calculations performed in the approximation of vanishing fermion masses which allows one to perform calculations with much less numerical efforts. Our results may be considered as part of the complete $O(\alpha)$ corrections which we will need to understand physics at future high energy linear colliders in a precise manner. The calculation of the missing virtual corrections to $e^+e^- \to 4\mathrm{f}$ is one of the big challenges for the future.

Acknowledgements. We thank Anja Werthenbach for carefully reading the manuscript.

References

1. F. Richard, J. R. Schneider, D. Trines, A. Wagner, "TESLA Technical Design Report Part I: Executive Summary," hep-ph/0106314
2. J. A. Aguilar-Saavedra et al. [ECFA/DESY LC Physics Working Group Collaboration], "TESLA Technical Design Report Part III: Physics at an e^+e^- Linear Collider", hep-ph/0106315
3. T. Abe et al. [American Linear Collider Working Group Collaboration], SLAC-R-570 Resource book for Snowmass 2001
4. K. Abe et al., hep-ph/0109166
5. D. Abbaneo et al. [LEP Collaborations], hep-ex/0103048
6. F. A. Berends, R. Pittau, R. Kleiss, Comput. Phys. Commun. **85**, 437 (1995) [hep-ph/9409326]
7. A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, Nucl. Phys. B **560**, 33 (1999) [hep-ph/9904472]
8. F. Jegerlehner, K. Kołodziej, Eur. Phys. J. C **12**, 77 (2000) [hep-ph/9907229]
9. F. Jegerlehner, K. Kołodziej, Eur. Phys. J. C **20**, 227 (2001) [hep-ph/0012250]
10. A. Vicini, Acta Phys. Polon. B **29**, 2847 (1998); hep-ph/0104164
11. A. Aeppli, D. Wyler, Phys. Lett. B **262**, 125 (1991)
12. A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, Nucl. Phys. B **587**, 67 (2000) [hep-ph/0006307]
13. W. Płaczek, S. Jadach, M. Skrzypek, B. F. Ward, Z. Wąs, hep-ph/0012094
14. J. Fleischer, F. Jegerlehner, K. Kołodziej, G. J. van Oldenborgh, Comput. Phys. Commun. **85**, 29 (1995) [hep-ph/9405380]
15. M. Böhm et al., Nucl. Phys. B **304**, 463 (1988); J. Fleischer, F. Jegerlehner, M. Zrałek, Z. Phys. C **42**, 409 (1989)

16. D.Yu. Bardin, S. Riemann, T. Riemann, Z. Phys. C **32**, 121 (1986); F. Jegerlehner, Z. Phys. C **32**, 425 (1986); A. Denner, T. Sack, Z. Phys. C **46**, 653 (1990)
17. H. Tanaka, T. Kaneko, Y. Shimizu, Comput. Phys. Commun. **64**, 149 (1991); S. Kawabata, Comput. Phys. Commun. **88**, 309 (1995); J. Fujimoto et al., Comput. Phys. Commun. **100**, 128 (1997) [hep-ph/9605312]; F. Yuasa et al., Prog. Theor. Phys. Suppl. **138**, 18 (2000) [hep-ph/0007053]
18. T. Stelzer, W. F. Long, Comput. Phys. Commun. **81**, 357 (1994) [hep-ph/9401258]; H. Murayama, I. Watanabe, K. Hagiwara, KEK-91-11
19. E. E. Boos, M. N. Dubinin, V. A. Ilin, A. E. Pukhov, V. I. Savrin, hep-ph/9503280; A. Pukhov et al., hep-ph/9908288; see also http://theory.npi.msu.su/~comphep
20. E. Accomando, A. Ballestrero, Comput. Phys. Commun. **99**, 270 (1997) [hep-ph/9607317]
21. F. A. Berends, C. G. Papadopoulos, R. Pittau, hep-ph/0002249; Comput. Phys. Commun. **136**, 148 (2001) [hep-ph/0011031]
22. A. Kanaki, C. G. Papadopoulos, Comput. Phys. Commun. **132**, 306 (2000) [hep-ph/0002082]; C. G. Papadopoulos, Comput. Phys. Commun. **137**, 247 (2001) [hep-ph/0007335]
23. G. Montagna, M. Moretti, O. Nicrosini, M. Osmo, F. Piccinini, Eur. Phys. J. C **21**, 291 (2001) [hep-ph/0103155]
24. F. Caravaglios, M. Moretti, Phys. Lett. B **358**, 332 (1995) [hep-ph/9507237]; Z. Phys. C **74**, 291 (1997) [hep-ph/9604316]
25. M. Moretti, T. Ohl, J. Reuter, hep-ph/0102195, http://heplix.ikp.physik.tu-darmstadt.de/~ohl/omega/; W. Kilian, (to be published), http://www-ttp.physik.uni-karlsruhe.de/~kilian/whizard/; T. Ohl, hep-ph/0011287
26. M.W. Grünewald et al., “Four-Fermion Production in Electron-Positron Collisions,” hep-ph/0005309
27. A. Denner, S. Dittmaier, M. Roth, D. Wackeroth, Eur. Phys. J. C **20**, 201 (2001) [hep-ph/0104057]
28. G. Montagna, M. Moretti, O. Nicrosini, M. Osmo, F. Piccinini, Phys. Lett. B **515**, 197 (2001) [hep-ph/0105120]
29. D. E. Groom et al. [Particle Data Group Collaboration], Eur. Phys. J. C **15**, 1 (2000)
30. G. P. Lepage, J. Comput. Phys. **27**, 192 (1978)
31. A. Biernacik, K. Ciećkiewicz, K. Kołodziej, Eur. Phys. J. C **20**, 233 (2001) [hep-ph/0102253]